Endogenous technological change

- Romer's (1986, 1990) key observation:

  ↳ NON-RIVALRY OF IDEAS

  ↳ HENCE, IDEAS ARE A NATURAL CANDIDATE TO BE A SOURCE OF INCREASING RETURNS.

- Consider $Y = F(A, K, L)$ with CRS with respect to K and L

  $$F(A, \lambda K, \lambda L) = \lambda F(A, K, L).$$

  Given that if $A_1 < A_2$, then $F(A_1, K, L) < F(A_2, K, L)$, we obtain:

  $$F(\lambda A, \lambda K, \lambda L) > \lambda F(A, K, L) \quad \text{for any } \lambda > 1.$$

- How to model endogenous technological change?

  ↳ THE BASIC IDEA IS SIMPLE

  ↳ THE ULTIMATE SOURCE OF LONG-RUN GROWTH IS SHIFTED TO THE R&D SECTOR

  ↳ IN A CLOSED ECONOMY, INNOVATIONS CREATED BY R&D TRANSLATE INTO THE ONLY SOURCE OF INCREASES IN "A"

  ↳ "A" IS TOTAL FACTOR PRODUCTIVITY / SOLOW RESIDUAL / "TECHNOLOGY LEVEL".

  ↳ HOWEVER, FINDING THE MARKET EQUILIBRIUM CAN BE TEDIOUS.

  → Social planner allocation (without R&D externalities)

  → INCREASING VARIETY MODELS (e.g., Romer 1990)

  → QUALITY LADDER "SCHUMPETERIAN" GROWTH MODELS (e.g., Aghion and Howitt 1992)

"Bare-bones" R&D-based growth model:

$$\begin{cases} Y = F(K,L) = A^{\sigma} K^{\alpha} L_Y^{1-\alpha} \\ \dot{K} = Y - C - \delta K = sY - \delta K & \leftarrow \text{capital's equation of motion} \\ \dot{A} = \gamma \cdot L_A^{\lambda} A & \leftarrow \text{R\&D equation} \\ L = L_A + L_Y, \quad L \equiv \text{const.} \end{cases}$$

(L_A – researchers, L_Y – other workers)

The dynamics may be complicated, but the first key observation follows directly from analyzing the balanced growth path (BGP)

- Assumption: at the BGP, all variables grow at a fixed rate.

(1) $\hat{A} = \frac{\dot{A}}{A} = \gamma L_A^{\lambda}$

(2) $\hat{Y} = \frac{\dot{Y}}{Y} = \sigma \hat{A} + \alpha \hat{K} + (1-\alpha)\hat{L}_Y$

(3) Assume $s \equiv \text{const}$, $\frac{L_A}{L}$ and $\frac{L_Y}{L}$ are constant as well (and thus L_A, L_Y)

(4) $\hat{K} = \frac{Y}{K} - \frac{C}{K} - \delta = s\frac{Y}{K} - \delta$

→ Hence $\frac{Y}{K} \equiv \text{const}$, implying $\hat{Y} = \hat{K} = \hat{C} = g$.

→ From (2) we have $\hat{K} = \sigma \hat{A} + \alpha \hat{K} \Rightarrow \hat{K} = g = \frac{\sigma}{1-\alpha}\hat{A}$.

Finally, the growth rate at BGP is:

$$\underbrace{g = \frac{\sigma}{1-\alpha}\gamma L_A^{\lambda}}$$

$\boxed{!}$

- the greater is L_A, the faster is growth
- note that in the optimal allocation, there will still be $L_Y > 0$ because it is required to generate immediate output (and consumption).

What is the "bare-bones" missing?

- DYNAMICS OUTSIDE OF BGP
- ENDOGENIZATION OF s, L_A (CHOICE VARIABLES) → soon!!

- INTERNATIONAL TECHNOLOGY DIFFUSION, IMITATION
- "TECHNOLOGY" CAN IN FACT BE MULTI-DIMENSIONAL!
  - investment-specific TC,
  - capital- vs. labor-augmenting TC, → needs to go beyond Cobb-Douglas technology.
  - vintage capital / human capital theory (embodied TC),
  - appropriate technology / world technology frontier (technologies suited to any given input mix),
  - there can be spillovers between various R&D sectors,
  - technology complexity / skill-biased TC.

<u>Empirical perspective on TFP</u>, $\boxed{A = \frac{Y}{K^\alpha L^{1-\alpha}}}$ (or $A_h = \frac{Y}{K^\alpha H_Y^{1-\alpha}}$?)

- the Solow residual is the "measure of our ignorance" (includes <u>everything but</u> factor inputs)
  - MISMEASUREMENT
  - PRODUCTION FUNCTION MISSPECIFICATION
  - ANY NON-NEUTRAL COMPONENT OF TC IS STILL REPORTED IN "A".

SOURCE: HOEG (2005)

- <u>Growth accounting</u>

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

→ contributes ≈ 15–40% of total variance in GDP/cap. growth rates across countries.

- <u>Levels accounting</u> (development accounting)

$$\ln\left(\frac{Y_i}{Y_0}\right) = \ln\left(\frac{A_i}{A_0}\right) + \alpha \ln\left(\frac{K_i}{K_0}\right) + (1-\alpha)\ln\left(\frac{L_i}{L_0}\right)$$

→ contributes ≈ 50–65% of total variance in GDP/cap. levels across countries.

How to measure "technological change"?

- TFP
- "purified" TC measures
  - → NET OF TECHNICAL EFFICIENCY CHANGES (WTF APPROACH)
  - → NET OF CAPACITY UTILIZATION
  - → ACCOUNTING FOR HUMAN CAPITAL ACCUMULATION

- direct measures of R&D (output / inputs)
  - ↳ PATENTS FILED/GRANTED (e.g., Madsen 2008)
  - ↳ PATENT CITATIONS (e.g., Hall, Jaffe, Trajtenberg 2001)

  - ↳ R&D SPENDING
  - ↳ R&D EMPLOYMENT / R&D SHARE IN EMPLOYMENT

Unit factor productivities

- Let's relax the assumption of a Cobb-Douglas production function
- E.g., CES technology:

$$\underbrace{Y = \left(\pi\,(A_K K)^{\frac{\sigma-1}{\sigma}} + (1-\pi)(A_L L)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}$$

- There is no unique "TFP"!
- The direction of TC (Acemoglu, 2003) $\boxed{!}$
  - PURELY LATC: $\hat{A}_K = 0$, $\hat{A}_L > 0$
  - PURELY KATC: $\hat{A}_L = 0$, $\hat{A}_K > 0$
  - A MIXTURE OF BOTH.

- The distribution of A_K and A_L across the world
  - ↳ Caselli & Coleman's (2006) take at the "world technology frontier".

Increasing variety models

↳ INTERMEDIATE GOODS (Romer, 1990) !

- "division of labor", process innovation

↳ FINAL GOODS (Grossman & Helpman, 1991)

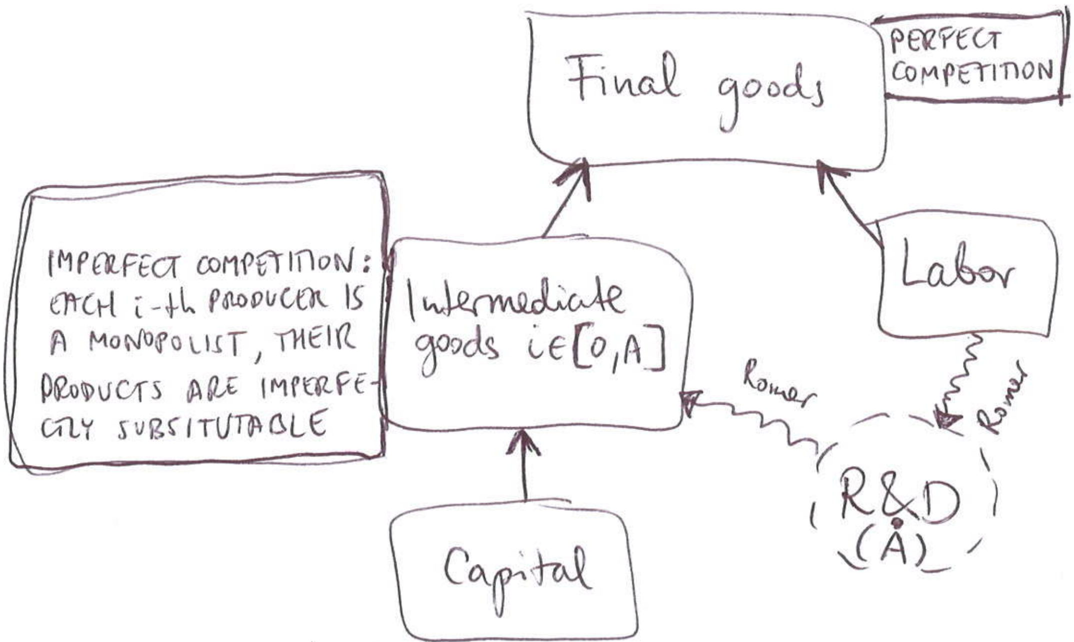
- "love of variety" preferences, product innovation

Quality ladders models

↳ SCHUMPETERIAN ("CREATIVE DESTRUCTION") MODELS (Aghion & Howitt, 1992)

- innovations increasing the quality of products, introducing new vintages.

Dixit & Stiglitz (1977) monopolistic competition model



- $\underbrace{Y = X^\alpha L_Y^{1-\alpha}}_{\text{FINAL GOOD PROD. FUNCTION}}$, where $\underbrace{x_i = k_i}_{\text{1-1 PROD. FUNCTION}}$ (for all $i\in[0,A]$) and $\underbrace{X = \left(\int_0^A x_i^\theta \, di\right)^{\frac{1}{\theta}}}_{\text{CES AGGREGATOR OF INTERMEDIATES}}$, $\theta\in(0,1)$.

1°/ Final goods* producers' problem

$$\max_{\{x_i\}, L_Y} \left\{ Y - \int_0^A q_i x_i \, di - wL \right\} \quad \text{where} \quad Y = \left(\int_0^A x_i^{\theta} \, di \right)^{\frac{\alpha}{\theta}} L_Y^{1-\alpha}$$

$$\frac{\partial \Pi}{\partial L_Y} = (1-\alpha)\frac{Y}{L_Y} - w = 0 \Rightarrow \underbrace{w = (1-\alpha)\frac{Y}{L_Y}}$$

$$\frac{\partial \Pi}{\partial x_i} = \frac{\alpha}{\theta}\left(\int_0^A x_i^{\theta} \, di \right)^{\frac{\alpha}{\theta}-1} \cdot L_Y^{1-\alpha} \, \theta x_i^{\theta-1} - q_i = 0$$

$$\Rightarrow q_i = \alpha \frac{Y}{X^{\theta}} x_i^{\theta-1} \Rightarrow \underbrace{x_i(q_i) = \left(\frac{\alpha Y}{q_i X^{\theta}} \right)^{\frac{1}{1-\theta}}}_{\text{DEMAND CURVE FOR INTERMEDIATE GOODS}}$$

2°/ Intermediate goods producers' problem $(i \in [0, A])$

$$\max_{q_i} \left\{ q_i x_i - \tilde{r} k_i \right\},$$ with $x_i = k_i$ and using the demand curve above, we obtain:

$$\max_{q_i} \left\{ (q_i - \tilde{r}) \, x_i(q_i) \right\}$$

$$\frac{\partial \pi_i}{\partial q_i} = x_i(q_i) + (q_i - \tilde{r}) \, x_i'(q_i) = x_i(q_i) + (q_i - \tilde{r})\left(\frac{-1}{1-\theta} \right) \frac{x_i(q_i)}{q_i} =$$

$$= \underbrace{x_i(q_i)}_{>0} \left(1 - \frac{1}{1-\theta} \frac{q_i - \tilde{r}}{q_i} \right) = 0.$$

Hence we have: $\frac{1}{1-\theta}\frac{q_i - \tilde{r}}{q_i} = 1 \Rightarrow 1 - \frac{\tilde{r}}{q_i} = 1 - \theta \Rightarrow \underbrace{q_i = \frac{\tilde{r}}{\theta}}$.

$$\left(\underbrace{q_i}_{\text{MONOPOLY PRICE}} = \underbrace{\frac{1}{\theta}}_{\text{CONSTANT MARKUP (\%)}} \cdot \underbrace{\tilde{r}}_{\text{MARGINAL COST OF PRODUCTION}} \right)$$

3°/ Symmetry of intermediate goods producers

$$q_i = \frac{\tilde{r}}{\theta} \quad \text{for all } i \in [0, A],$$

hence $\underbrace{q_i = q_j = \bar{q}}$, implying $x_i = \bar{x}$ for all i.

Monopoly profits are equal to:

$$\bar{\pi} = (\bar{q} - \tilde{r})\bar{x} = \left(\frac{\tilde{r}}{\theta} - \tilde{r}\right)\bar{x} = \left(\frac{1-\theta}{\theta}\right)\tilde{r}\bar{x}.$$

4°/ General equilibrium (so far)

a) $$K = \int_0^A k_i \, di = \int_0^A x_i \, di = \int_0^A \bar{x} \, di = A\bar{x}, \quad \text{hence } \underbrace{\bar{x} = \frac{K}{A}}.$$

b) $$X = \left(\int_0^A \bar{x}^{\theta} di\right)^{\frac{1}{\theta}} = \left(A\bar{x}^{\theta}\right)^{\frac{1}{\theta}} = A^{\frac{1}{\theta}}\bar{x} = A^{\frac{1}{\theta}}\frac{K}{A} = A^{\frac{1-\theta}{\theta}}K.$$

c) $$Y = X^{\alpha} L_Y^{1-\alpha} = \underbrace{\overbrace{A^{\frac{\alpha(1-\theta)}{\theta}}}_{\text{technology}} \underbrace{K^{\alpha} L_Y^{1-\alpha}}_{\text{CRS}}}_{\text{IRS}}.$$

d) $$\bar{q} = \frac{\tilde{r}}{\theta} = \alpha \frac{Y}{X^{\theta}}\bar{x}^{\theta-1} = \alpha \frac{Y}{\cancel{\bar{x}^{\theta}}A} \cdot \cancel{\bar{x}^{\theta}} \cdot \frac{1}{\bar{x}} = \alpha \frac{Y}{K} \Rightarrow \underbrace{\tilde{r} = \alpha\theta\frac{Y}{K}}$$

e) $$\int_0^A \pi_i \, di = \int_0^A \bar{\pi} \, di = A\bar{\pi} = \left(\frac{1-\theta}{\theta}\right)\tilde{r}A\bar{x} = \alpha(1-\theta)Y$$

f) final output is divided according to:

$$\tilde{r}K + wL_Y + \int_0^A \pi_i \, di = \underbrace{\alpha\theta Y}_{\tilde{r}K} + \underbrace{(1-\alpha)Y}_{wL_Y} + \underbrace{\alpha(1-\theta)Y}_{\text{profits.}} = Y$$

5°/ <u>Households</u> — <u>DYNAMIC</u> optimization problem

$$\max \int_0^\infty e^{-\rho t} u(c)\,dt \quad \text{s.t.} \quad \dot a = ra + w - c,$$

yielding $\underbrace{\hat c = \frac{r-\rho}{\theta}}_{\text{EULER EQ.}}$ (with CRRA preferences).

- <u>Assets</u> are kept in the form of
  (a) capital,
  (b) shares of intermediate goods firms.

- Assets equation (capital market clearing): $a = k + p_A \frac{A}{L}$.

- Assets have equal returns:

NET RETURN

$$\frac{\dot p_A}{p_A} + \frac{\pi_i}{p_A} = r \iff r p_A = \underbrace{\pi_i}_{\text{DIVIDEND}} + \underbrace{\dot p_A}_{\text{RESALE VALUE}}.$$

- Capital follows:

$$\dot k = y - c - \delta k.$$

6/ R&D firms

- create new varieties of intermediate goods,
- sell the right to produce (patents) to households,
- patents have infinite duration and patent protection is perfect,
- there is free entry to R&D,
- growth effects of innovations are not internalized by R&D firms.

→ $\max\limits_{L_A} \{ p_A \underbrace{\bar{\nu} L_A}_{\dot{A}} - w L_A \}$, taking p_A, $\bar{\nu}$, w as given.

R&D EXTERNALITY

By free entry, $\underline{w = p_A \bar{\nu}}$.

R&D DUPLICATION EXTERNALITY

GROWTH EFFECT $\boxed{!!}$

→ Upon aggregation, $\dot{A} = \gamma L_A^{\lambda} A = \underbrace{\gamma \cdot L_A^{\lambda-1} A}_{\bar{\nu}} L_A$

7/ Labor market clears: $L = L_A + L_Y$, equal wage.

$$w = (1-\alpha)\frac{Y}{L_Y} = p_A \gamma L_A^{\lambda-1} A.$$

$$\left(\text{Hence, } p_A = \frac{(1-\alpha) Y}{L_Y \gamma L_A^{\lambda-1} A} \right).$$

8°/ Dynamics (4 key variables: $\underbrace{k, A}_{\text{STATE VARS}}; \underbrace{c, L_A}_{\text{CONTROL VARS}}$)

$$\begin{cases} \hat{c} = \frac{1}{\theta}(r-\rho) = \frac{1}{\theta}\left(\alpha\theta \frac{y}{k} - \delta - \rho\right) \\ \hat{k} = \frac{y}{k} - \frac{c}{k} - \delta \\ \hat{A} = \gamma L_A^{\lambda} \\ y = A^{\frac{\alpha(1-\theta)}{\theta}} \cdot k^{\alpha}\left(\frac{L_Y}{L}\right)^{1-\alpha} \\ \hat{p}_A = r - \frac{\pi_i}{p_A} \quad \text{(capital market)} \\ p_A = \frac{(1-\alpha)Y}{L_Y \gamma L_A^{\lambda-1} A} \quad \text{(labor market)} \\ L_Y = L - L_A \end{cases}$$

→ ONE CAN ELIMINATE p_A, GET DYNAMICS OF L_A, L_Y.

[See Arnold(2006)].

9°/ BGP equilibrium

- $\frac{y}{k} \equiv$ const, $\frac{c}{k} \equiv$ const, $\hat{c} = \hat{k} = \hat{y} := g$ ← GROWTH RATE
- $\frac{L_A}{L} \equiv$ const, $\frac{L_Y}{L} \equiv$ const, constant savings rate; $L_A^* \equiv$ const, $L_Y^* \equiv$ const.
- $\hat{A} = \gamma L_A^{*\lambda}$
- $\hat{y} = \frac{\alpha(1-\theta)}{\theta}\hat{A} + \alpha\hat{y} \Rightarrow \underbrace{\hat{y} = \overset{\sigma}{\frac{\alpha(1-\theta)}{\theta}} \cdot \frac{1}{1-\alpha} \cdot \gamma L_A^{*\lambda}}$

$$\boxed{g = \frac{\alpha}{1-\alpha} \cdot \frac{1-\theta}{\theta} \cdot \gamma L_A^{*\lambda}}$$

GROWTH RATE OF THE ECONOMY

$$\begin{cases} \hat{p}_A = \hat{Y} - \hat{A} = g - \frac{1-\alpha}{\alpha}\frac{\theta}{1-\theta} g . \\ \hat{p}_A = \underbrace{\alpha\theta \frac{y}{k} - \delta}_{r} - \underbrace{\alpha(1-\theta)\frac{\cancel{Y}}{\cancel{A}}}_{\pi_i} \cdot \underbrace{\frac{L_Y \gamma L_A^{\lambda-1} \cancel{A}}{(1-\alpha)\cancel{Y}}}_{1/p_A} = \underbrace{\left(\alpha\theta \frac{y}{k} - \delta\right)}_{r} - \frac{\alpha(1-\theta)}{1-\alpha}\frac{L_Y}{L_A}\underbrace{\left(\gamma L_A^{\lambda}\right)}_{\hat{A}} \end{cases}$$

with $\hat{c} = \frac{r-\rho}{\theta} \Rightarrow r = \theta g + \rho$; $\theta\hat{c} = \alpha\theta\frac{y}{k} - \delta - \rho$

$$\Rightarrow \boxed{\frac{y}{k} = \frac{\theta g + \delta + \rho}{\alpha\theta}}$$

Hence, along the BGP:

$$g - \hat{A} = \theta g + \rho - \frac{\alpha}{1-\alpha}(1-\theta)\frac{L_Y}{L_A}\hat{A}$$

$$-(1-\theta)g + \hat{A} + \rho = + \frac{\alpha}{1-\alpha}(1-\theta)\frac{L_Y}{L_A}\hat{A}$$

$$\frac{1-L_A^*}{L_A^*} = \frac{L_Y^*}{L_A^*} = \frac{\hat{A} + \rho - (1-\theta)g}{(1-\theta)\hat{A}} \cdot \frac{1-\alpha}{\alpha} = \left(\frac{1-\alpha}{\alpha}\right)\cdot\left(\frac{1}{1-\theta} + \rho \cdot \frac{\alpha}{1-\alpha}\cdot\frac{1-\theta}{\theta}\cdot\frac{1}{g} - \frac{\alpha}{1-\alpha}\cdot\frac{1-\theta}{\theta}\right)$$

↳ the equilibrium allocation of labor between L_A & L_Y (R&D and output).

↳ depends on: α, θ, ρ ; $\underbrace{\gamma, \lambda}_{\text{R\&D TECHNOLOGY}}$.

α: TECHNOLOGY
θ: MARKUP PARAMETER
ρ: IMPATIENCE.